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Mark Scheme (Results)

June 2011

GCE Core Mathematics C3 (6665) Paper 1

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

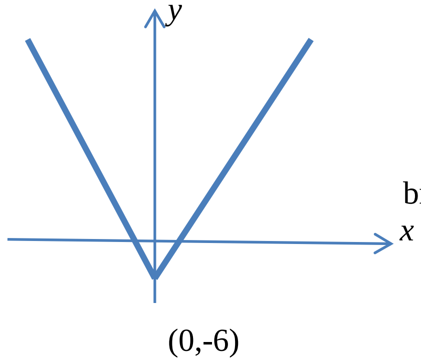
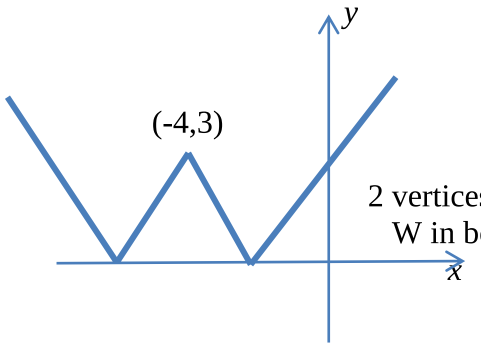
   - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - B marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - $\square$ The second mark is dependent on gaining the first mark

Question Number	Scheme	Marks
1 (a)	$\frac{1}{(x^2+3x+5)} \times \ldots$, = $\frac{2x+3}{(x^2+3x+5)}$	M1,A1 (2)
(b)	Applying $\frac{vu'-uv'}{v^2}$ $\frac{x^2\times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2\sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	M1, A2,1,0 (3) 5 Marks
2 (a)	f(0.75)= -0.18… f(0.85) = 0.17….. Change of sign, hence root between x=0.75 and x=0.85	M1 A1 (2)
(b)	Sub x_0=0.8 into $x_{n+1} = [\arcsin(1-0.5x_n)]^{\frac{1}{2}}$ to obtain x_1 Awrt x_1=0.80219 and x_2=0.80133 Awrt x_3 = 0.80167	M1 A1 A1 (3)
(c)	f(0.801565) = -2.7…..$\times 10^{-5}$ f(0.801575) = +8.6….$\times 10^{-6}$ Change of sign and conclusion See Notes for continued iteration method	M1A1 A1 (3) 8 Marks

Question Number	Scheme	Marks
3 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	'y' co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R'=(-4,3)	B1
		(3)
		6 Marks
4 (a)	$y = 4 - \ln(x+2)$	
	$\ln(x+2) = 4 - y$	
	$x + 2 = e^{4-y}$	
	$x = e^{4-y} - 2$	M1
	$f^{-1}(x) = e^{4-x} - 2$ oe	M1A1
		(3)
(b)	$x \leq 4$	B1
		(1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$	M1
	$fg(x) = 4 - x^2$	dM1A1
		(3)
(d)	$fg(x) \leq 4$	B1ft
		(1)
		8 Marks

Question Number	Scheme	Marks
5 (a)	p=7.5	B1 (1)
(b)	$2.5 = 7.5e^{-4k}$	M1
	$e^{-4k} = \frac{1}{3}$	M1
	$-4k = \ln(\frac{1}{3})$	dM1
	$-4k = -\ln(3)$	
	$k = \frac{1}{4}\ln(3)$	A1*
	See notes for additional correct solutions and the last A1	(4)
(c)	$\frac{dm}{dt} = -kpe^{-kt}$ ft on their p and k	M1A1ft
	$-\frac{1}{4}ln3 \times 7.5e^{-\frac{1}{4}(ln3)t} = -0.6ln3$	
	$e^{-\frac{1}{4}(ln3)t} = \frac{2.4}{7.5} = (0.32)$	M1A1
	$-\frac{1}{4}(ln3)t = \ln(0.32)$	dM1
	t=4.1486…. 4.15 or awrt 4.1	A1
		(6)
		11Marks

Question Number	Scheme	Marks
6 (a)	$\dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} = \dfrac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$= \dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$	M1A1
	$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta$ cso	A1* (4)
(b)(i)	$\tan 15° = \dfrac{1}{\sin 30°} - \dfrac{\cos 30°}{\sin 30°}$	M1
	$\tan 15° = \dfrac{1}{\frac{1}{2}} - \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ cso	dM1 A1* (3)
(b)(ii)	$\tan 2x = 1$	M1
	$2x = 45°$	A1
	$2x = 45° + 180°$	M1
	$x = 22.5°, 112.5°, 202.5°, 292.5°$	A1(any two) A1 (5)
		12 Marks
	Alt for (b)(i) $\tan 15° = \tan(60° - 45°)$ *or* $\tan(45° - 30°)$	
	$\tan 15° = \dfrac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ *or* $\dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$	M1
	$\tan 15° = \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}$ *or* $\dfrac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$	M1
	Rationalises to produce $\tan 15° = 2 - \sqrt{3}$	A1*

Question Number	Scheme	Marks
7 (a)	$x^2-9=(x+3)(x-3)$	B1
	$\dfrac{4x-5}{(2x+1)(x-3)}-\dfrac{2x}{(x+3)(x-3)}$	
	$=\dfrac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)}-\dfrac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$=\dfrac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$=\dfrac{5(x-3)}{(2x+1)(x-3)(x+3)}=\dfrac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f(x)=\dfrac{5}{2x^2+7x+3}$	
	$f'(x)=\dfrac{-5(\mathbf{4x+7})}{(2x^2+7x+3)^2}$	M1**M1**A1
	$f'(-1)=-\dfrac{15}{4}$	M1A1
	Uses $m_1m_2=-1$ to give gradient of normal$=\frac{4}{15}$	M1
	$\dfrac{y-(-\frac{5}{2})}{(x--1)}=their\ \dfrac{4}{15}$	M1
	$y+\frac{5}{2}=\frac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		13 Marks

Question Number	Scheme	Marks
8		
(a)	$R^2 = 2^2 + 3^2$	M1
	$R = \sqrt{13}$ or 3.61 …..	A1
	$\tan\alpha = \frac{3}{2}$	M1
	$\alpha = 0.983 \ldots$	A1
		(4)
(b)	$f'(x) = 2e^{2x}\cos3x - 3e^{2x}\sin3x$	M1A1A1
	$= e^{2x}(2\cos3x - 3\sin3x)$	M1
	$= e^{2x}(R\cos(3x + \alpha)$	
	$= Re^{2x}\cos(3x + \alpha)$	A1* cso
		(5)
(c)	$f'(x) = 0 \Rightarrow \cos(3x + \alpha) = 0$	M1
	$3x + \alpha = \frac{\pi}{2}$	M1
	x=0.196… awrt 0.20	A1
		(3)
		12 Marks
	Alternative to part (c)⇒	
	$f'(x) = 0 \Rightarrow 2\cos3x - 3\sin3x = 0$	M1
	$\tan3x = \frac{2}{3}$	M1
	x=0.196… awrt 0.20	A1
		(3)

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June 2011 C3 6665/01Additional notes/guidance for examiners Pre standardisation version

1 (a) M1 Use of chain rule on ln(x^2+3x+5).

Be generous $\frac{1}{x^2 \ \ +3x+5} \times (something)$ is sufficient for this.

The something could be 1 BUT not 0

A1 Correct answer. Accept $\frac{2x+3}{x^2 \ \ +3x+5}$, $\frac{1}{x^2 \ \ +3x+5} \times (2x+3)$

NB. $\frac{1}{x^2 \ \ +3x+5} \times 2x+3$ is NOT sufficient for the final A1. This scores M1A0

1(b) M1 Use of quotient rule, a form of which appears in the formula booklet. I**f the formula is quoted**, it **must be correct** and an **attempt must** be made at both differentials.

If the formula is not quoted then you **mus**t see an expression of the form

$$\frac{x^2(\pm sinx) - cosx \times 2x}{(x^2)^2}$$

The absence of/ or incorrect bracketing can be ignored for this mark

A1 A correct numerator or a correct denominator. Note that M1 must have been scored.

Acceptable forms for the numerator are $\boldsymbol{x^2(-sinx) - cosx \times 2x}$ or $\boldsymbol{x^2. -sinx - cosx.2x}$

Bracketing must be correct but do not penalise if a correct final answer is given

Acceptable forms for the denominator are $(x^2)^2$ and x^4

A1 A correct expression for the differential. The question did not ask for simplification

Accept forms such as $\frac{x^2(-sinx)-cosx\times 2x}{x^4}$, $\frac{x^2.-sinx-cosx.2x}{x^4}$, $\frac{x^2.-sinx-cosx.2x}{(x^2)^2}$, $\frac{-\text{xsin}x-2\text{cos}x}{x^3}$

Alternative to (b)

M1 Writing $\frac{cosx}{x^2}$ as $\cos x \times x^{-2}$ and applying the product rule. I**f the formula is quoted**, it **must be correct** and an **attempt must** be made at both differentials. If the formula is not quoted you must see an expression of the form $x^{-2} \times \pm sinx + cosx \times \pm \cdots x^{-3}$

A1 One term of $x^{-2} \times -sinx + cosx \times -2x^{-3}$

A1 Both terms correct Eg. $x^{-2} \times -sinx + cosx \times -2x^{-3}$

2.(a) M1 Calculates f(0.75) AND f(0.85) with at least one correct to 1sf (rounded or truncated)

A1 Both correct to 1 sf (rounded or truncated), reason and minimal conclusion.

f(0.75)= awrt-0.2 rounded or -0.1 truncated. f(0.85)=awrt 0.2 rounded or 0.1 truncated.

Accept change of sign, hence root. Accept $> 0, < 0, therefore\ true$.

NB. A smaller interval is acceptable but the question specifically states [0.75,0.85] so for the M1 (and therefore A1) mark the conclusion must refer back to that interval.

x	f(x)
0.75	-0.18339
0.76	-0.14797
0.77	-0.11246
0.78	-0.07689
0.79	-0.04127
0.8	-0.00561
0.81	0.030062
0.82	0.065731
0.83	0.101377
0.84	0.136981
0.85	0.172524

(b) M1 Substitutes x_0=0.8 into the iterative formula to evaluate x_1.

$\sqrt{arcsin}0.6$,awrt 0.8 or 6(°) are examples.

A1 Awrt x_1=0.80219, and x_2=0.80133

A1 Awrt x_3=0.80167

NB. The suffixes are not important. Mark successive answers which could appear as x_2 x_3 x_4

Note that this appears as B1,B1,B1 e-pen

(c) M1 Chooses the end points of the interval [0.801565,0.801575], or any tighter interval that contains the accurate answer 0.8015726 **AND** attempts f(x) for both**.**

A1 Calculates correctly to one figure (rounded or truncated) both f(0.801565) and f(0.801575) or f(x) at the end points of an appropriately tighter interval.

f(0.801565)=awrt-0.00003 rounded or -0.00002 truncated

f(0.801575) =awrt 0.000009 rounded (accept 0.00001 which is correct to 5dp) or 0.000008 truncated

A1 Both previous marks must have been scored. A reason and (minimal) conclusion must be given. No reference to 5 dp needs to be made.

Acceptable reasons are 'change of sign' $> 0, < 0$ beside the answers, f(a).f(b)< 0

Acceptable conclusions are 'hence root', hence true, QED and □

Alternatives for (c).

ALT 1 M1 Chooses the end points of the interval [0.801565,0.801575], or any tighter interval that contains the accurate answer 0.8015726 AND attempts g(x) for both where g(x)=$\pm(\sqrt{(\arcsin(1-0.5x)} - x)$

A1 Calculates correctly to one figure (rounded or truncated) both g(0.801565) and g(0.801575) or g(x) at the end points of an appropriately tighter interval

Typically g(0.801565)=0.000010537 and g(0.801575)=-0.000003358

A1 Both previous marks must have been scored. **A reason which must make reference back to f(x) and not just g(x)** and (minimal) conclusion must be given. No reference to 5 dp needs to be made.

The question does not stipulate a method for (c) so continued iteration is acceptable **as the sequence oscillates**

ALT 2 M1 Continued iteration at least as far as x_7. Accept for this, statements such as .... x_{10}=0.80157

A1 Awrt x_4=0.801536, x_5=0.801587, x_6=0.801567 and x_7=0.801575 or any **two successive** values later in the sequence **given to at least 6dp**

A1 Both previous marks must have been scored. **A reason** which must state that all values (after x_7) round to 0.80157 **and a conclusion** which must state to 5 dp.

The table below shows values of x_0 to x_9 .

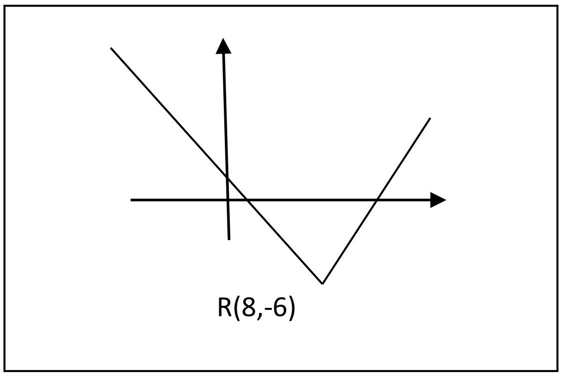
X_0	0.8
X_1	0.802185209
X_2	0.80133392
X_3	0.801665559
X_4	0.801536362
X_5	0.801586694
X_6	0.801567086
X_7	0.801574725
X_8	0.801571749
X_9	0.801572908

3. (a) B1 Generously award any 'V' shape. Its position does not matter

B1 The vertex is on the negative y axis. This is in effect awarding a mark for moving the graph 4 units to the left. Both branches of the graph must cross the x axis

B1 The **y coordinate** of 'R' is at -6. This is a mark for a stretch of 2 in the y direction. Accept (-6,0) written on the y axis. If in doubt send to review

Note: These marks are independent **but must be scored in this order on e- pen.**



For example this would score

B1 for the 'V' shape

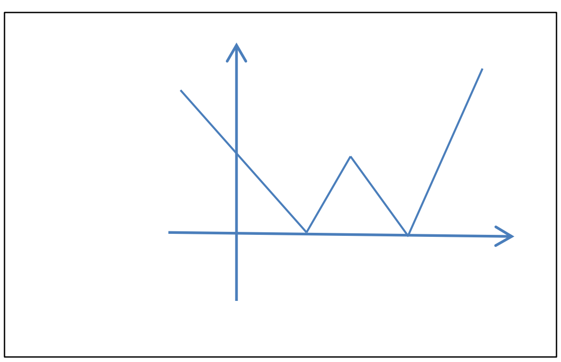
B0 vertex is not on the y axis

B1 vertex has a y co ordinate of -6

(b) B1 For a 'W' shape. Its position does not matter.

B1dep Two vertices on the negative x axis. The W appears in both quadrants 1 and 2. Ignore any co- ordinates for this mark.

B1 R' written/or marked on the graph/ or axes as (-4,3). **Do not award if extra R's are given**.



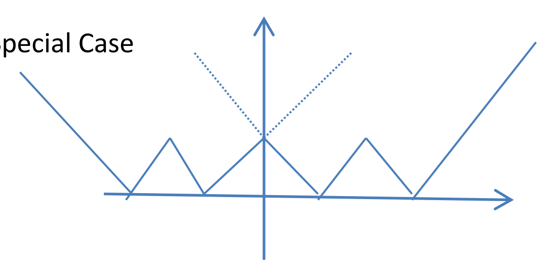
For example this (which is | -f(*x*) | would score

B1 W shape

B0 The two vertices are not on the negative x axis

B0 R' incorrect

Special Case



Two W's (with or without the dashed lines)

scores 1,0,0 special case

4.(a) M1 A generous mark for an attempt to make x or a swapped y the subject of the formula $y = 4 - \ln(x+2)$. It must be a full method and cannot finish x+2=f(y) but you can condone 'poor' algebra.

M1 For applying 'the order of operations' correctly **AND** knowing that e^x and ln(x) are inverse functions. Condone slips in signs here but you would expect to see

$\ln(x+2) = \pm y \pm 4$ going to $x + 2 = e^{\pm y \pm 4}$ for this mark.

Watch for $-\ln(x+2) = \pm y \pm 4$ going to $-(x+2) = e^{\pm y \pm 4}$ .This is M0-incorrect 'ln' work.

A1 Obtaining a correct expression for $f^{-1}(x)$. $Eg.\ f^{-1}(x) \to -2 + e^{4-x}$ or $-2 + e^{-(x-4)}$

It must be in terms of x, but could be expressed 'y=' or $f^{-1}: x \to$

(b) B1 Stating the correct domain.

Accept x≤4, (-∞,4], Domain is all values (of x) less than or equal to 4 **BUT do not accept y or f^{-1}≤4**

(c) M1 For an attempt to substitute g into f. Eg $4 - \ln(e^{x^2} - 2 + 2)$, $4 - \ln(e^{x^2} + 2 - 2)$.

A correct answer with no working (or indeed incorrect working) would imply this. Do not accept if there is obvious incorrect working. $4 - \ln(e^{x^2} - 2)$ is M0.

dM1 Dealing with the 'brackets' **first** and knowing that ln and exp are inverse fn's

Seeing $4 - \ln(e^{x^2})$ leading to 4- 'a function of x' scores this mark

Note: $4 - \ln(e^{x^2}) - ln2 + ln2 = 4 - x^2$ would lose this mark and score 1,0,0

This is dependent upon the first M mark being scored.

A1 cso $\{fg(x)\} = 4 - x^2$.

All 3 marks must be awarded if $4 - x^2$ is reached given that no incorrect working is seen

(d) B1ft Stating the correct range.

Accept y≤4, (-∞,4], Range is all values less than or equal to 4 **BUT do not accept x≤4**

This is a follow through on all answers in (b) apart from $x\mathcal{E}\mathcal{R}$ leading to y$\mathcal{E}\mathcal{R}$

5. For the purposes of marking (a) and (b) can be scored together

(a) B1 Stating that p= 7.5

(b) M1 Substituting both **m=2.5 and t=4** into $m = pe^{-kt}$ with 'their' p. Eg. $2.5 = pe^{-k4}$,

M1 Dividing by their 'p' thus arriving at the result $e^{\pm 4k} = A$ where A is a constant

dM1 Correct order of operations to make their '±4k' the subject by takings ln's. Eg. ±4k=lnA
This is dependent upon the second M mark

A1(*) cso. Completes proof and shows that $k = \frac{1}{4}ln3$.

Possibilities are $-4k = \ln\left(\frac{1}{3}\right) \Rightarrow -4k = -\ln(3) \Rightarrow k = \frac{1}{4}ln3.$
Or $-4k = \ln\left(\frac{1}{3}\right) \Rightarrow -4k = \ln(1) - \ln(3) \Rightarrow -4k = -\ln(3) \Rightarrow k = \frac{1}{4}ln3$

Note that $e^{-4k} = \frac{1}{3} \Rightarrow \; e^{4k} = 3 \Rightarrow 4\text{k} = \ln 3 \Rightarrow k = \frac{1}{4}ln3$ scores full marks
But $e^{-4k} = \frac{1}{3} \Rightarrow -4k = \ln\left(\frac{1}{3}\right) \Rightarrow k = \frac{1}{4}ln3$ scores 1,1,1,0

(c) M1 Differentiating their expression $m = pe^{\pm kt}$ to $\frac{dm}{dt} = Ce^{\pm kt}$ where C is a constant

A1ft Correct answer for $\frac{dm}{dt}$ following through on their numerical values of p and k.

For example $m = 7.5\, e^{\frac{1}{4}(ln3)t}$ leading to $\frac{dm}{dt} = \frac{7.5}{4} ln3e^{\frac{1}{4}(ln3)t}$ scores M1A1ft
Candidates can put answers in decimals so accept awrt $-2.06\, e^{-\frac{1}{4}(ln3)t}$

M1 Sets their dm/dt =-0.6 ln3, **and** rearranges **their** equation into $e^{\pm kt} = constant$

A1 $e^{-kt} = \frac{2.4}{7.5}$ or $\frac{8}{25}$ OR ANY EQUIVALENT **with** k= $\frac{1}{4}ln3$

dM1 Takes ln of both sides, leading to a solution of t. This is dependent on the previous M1.

A1 4.15 or awrt 4.1

Alt (b) Candidates can use the given value of $k = \frac{1}{4}ln3$ to prove that m=2.5 when t=4 (OR Similar Arguments)

M1 Substitutes t=4 and $k = \frac{1}{4}ln3$ into $m =' 7.5'e^{-kt}$ to get $'7.5'e^{-ln3}$

M1 Simplifies correctly $e^{-ln3} = \frac{1}{3}$ .Intermediate step of $\frac{1}{e^{ln3}}$ or $e^{ln3^{-1}} = 3^{-1}$ **is required**

dM1 Shows that m=2.5 .

A1 Gives **a minimal conclusion.** Hence $k = \frac{1}{4}ln3$ is acceptable.

Alt (b) Candidates take ln's of both sides

M1 Substituting both **m=2.5 and t=4** into $m = pe^{-kt}$ with 'their' p. Eg. $2.5 = pe^{-k4}$

M1 Take ln's of both sides **and** correctly use ln(ab) =lna+lnb law

$Eg. \ln(2.5) = \ln(7.5e^{-4k}) \Rightarrow \ln(2.5) = \ln(7.5) + \ln(e^{-4k})$

dM1 Correct order of operations to make '±4k' the subject by using lna-lnb rule

Eg $\ln\left(\frac{2.5}{7.5}\right) = \pm 4k$. This is dependent upon the second M mark.

A1(*) cso. Shows that $k = \frac{1}{4}ln3$.

6. (a) M1 Write as a single fraction. Candidates show that they need to combine the terms. Most often this will appear as $\frac{1-cos2\theta}{sin2\theta}$ but sometimes as $\frac{1-(cos^2\theta-sin^2\theta)}{2sin\theta cos\theta}$

M1 Use the **correct identity** for $sin2\theta = (2sin\theta cos\theta)$ **and** a **correct identity** for $cos2\theta = (cos^2\theta - sin^2\theta \; or \; 2cos^2\theta - 1 \; or \; 1 - 2sin^2\theta)$ **in the expression**

A1 Reach the expression $\frac{2sin^2\theta}{2sin\theta cos\theta}$

A1(*) Cancel down to show **both** $\frac{sin\theta}{cos\theta}$ (or $\frac{\cancel{2}sin^{\cancel{2}}\theta}{\cancel{2}\cancel{sin\theta}cos\theta}$) **and** tanθ.

Note: Mixed variables M1M1A0A0 if they use all the way through

M1M1A1A0 if they recover and write down $\frac{2sin^2\theta}{2sin\theta cos\theta}$ after both *x*'s and θs

(bi) M1 Substitute θ=15 into part (a) to reach $tan15 = \frac{1}{sin30} - \frac{cos30}{sin30}$

dM1 Uses both $sin30 = \frac{1}{2}$ and $cos30 = \frac{\sqrt{3}}{2}$ (or $sin30 = \frac{1}{2}$ and $tan30 = \frac{1}{\sqrt{3}}$) or ($sin30 = \frac{1}{2}$ and $\cot 30 = \sqrt{3}$) or $cosec30 = 2$ and $\cot 30 = \sqrt{3}$) in the expression.
This is dependent upon the first M1 being scored and is given for an intermediate line.

A1(*) Correct answer given. **Note that this is a given solution so check carefully.**

(bii) M1 For writing $tan\mathbf{2}x = 1$. Condone x being replaced by θ or other variables.

A1 Getting one solution of '2x', most likely to be 45(°)**or** one solution of x, likely to be 22.5(°)

M1 Achieving a secondary value of their '2x' (most likely 4x from a tan(4x)=1 equation, but it must be for a function of tan. Most likely to be 225 or 405. **This could be implied by a second correct answer**

A1 Any two values of 22.5,112.5,202.5,292.5. Accept equivalents like $\frac{45}{2}$ etc.Accept answers rounding to 23,113,203 and 293 as some candidates use the double angle tan substitution and solve a quadratic creating accuracy problems

A1 CAO ALL 4 values or equivalents 22.5,112.5,202.5,292.5 and no others inside the range

Notes: Ignore extra solutions outside the range. Radian answers score M1A0M1A1A1 for fully correct solutions of $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$. Two answers in degrees most often scores 4 out of 5 marks-m1a1m1a1a0

Candidates who write down some or all answers without working (or seemingly incorrect working) should be referred to your team leader via review.

Alt for (b). Question is a hence or otherwise so we accept for b(i)

M1 $\tan 15 = tan(45-30) = \frac{tan45-tan30}{1+tan45tan30}$, $\tan 15 = tan(60-45) = \frac{tan60-tan45}{1+tan60tan45}$. Formula must be correct.

dM1 Use the above with both $tan45 = 1$ and $tan30 = \frac{\sqrt{3}}{3}$ or $tan60 = \sqrt{3}$

A1(*) Correctly rationalises expression by multiplying both numerator and denominator by $1 - \frac{\sqrt{3}}{3}$ oe to produce given solution

7. (a) B1 Expresses $x^2 - 9$ as $(x+3)(x-3)$. This could appear later in the solution.

M1 Combines two fractions to form a single fraction.
Condone slips on numerator but **both** numerators must have been 'adapted'
Note that they could appear separately as $\frac{(4x-5)(x^2-9)}{(2x+1)(x^2-9)(x-3)} - \frac{2x(2x+1)(x-3)}{(x^2-9)(2x+1)(x-3)} -$

M1 The single fraction must be of the form $\frac{simplified\ numerator}{(2x+1)(x+3)(x-3)}$
Condone invisible brackets on denominator. If the original denominator was 'quartic' then (x-3) must be factorised out of both the numerator and the denominator. **The brackets on the numerator must have been removed and the terms simplified**

A1 Correct fraction of $\frac{5x-15}{(2x+1)(x+3)(x-3)}$ or $\frac{5x-15}{(2x+1)(x^2-9)}$ or $\frac{5x-15}{(2x^3+x^2-18x-9)}$ is obtained

A1(*) Correct solution only. The $(5x-15)$ **must** be seen to be factorised **and** the (x-3) terms cancelled to produce given answer

(b) M1 Using the chain rule or the quotient rule to deal with $\frac{5}{(2x+1)(x+3)}$
Chain rule look for $((2x+1)(x+3))^{-2} \times (\dots\dots)$ or $(their\ quadratic)^{-2} \times (\dots)$.NB (...)could be 1
Quotient rule look for $\frac{(2x+1)(x+3)\times 0 - 5(\dots\dots\dots)}{((2x+1)(x+3))^2}$. It is not acceptable to have 5 differentiating to be 1 in this method.

M1 The correct method of differentiating (2x+1)(x+3). It is independent and can be given for an attempt to multiply out and differentiating or use of product rule. Sight of 4x+7 is usually sufficient for this mark**. Note that this appears as A1 on epen**

A1 Correct (unsimplified) answer for f'(x) Eg. $\frac{(2x+1)(x+3)\times 0 - 5\times(4x+7)}{((2x+1)(x+3))^2}$ or $5((2x+1)(x+3))^{-2} \times (4x+7)$

M1 Substitutes x=-1 into their f'(x). Watch out for candidates who sub $x = -\frac{5}{2}$ which gets $f'(x) = \frac{15}{4}$.

A1 Gradient is $\frac{-15}{4}$ or equivalent (-3.75). **Do not accept just this answer for 5 marks! Send to review.**

M1 Applies $m_1 = -\frac{1}{m_2}$ for their tangent gradient. It must be numerical. It could appear later

M1 **Uses P and** an **adapted numerical tangent gradient** to find an equation of the normal.
Eg. $\left(y - -\frac{5}{2}\right) = m_1(x - -1)$. If formula quoted allow one slip on- $\frac{5}{2}$ or -1 in substitution.

A1 Any correct equation. Eg $y + \frac{5}{2} = \frac{4}{15}(x+1)$, $8x - 30y = 67$, $y = \frac{4}{15}x - \frac{67}{30}$
The negatives must have been dealt with
Note: Remember to 'isw' in part (b)

Alternative to 7(b) using partial fractions. First 3 marks are scored

M1 Writes f(x) in the form $\frac{A}{(2x+1)} + \frac{B}{(x+3)}$ using Partial Fractions. Note that A=2 ,B=-1

M1 Differentiates this expression to a form $\frac{C}{(2x+1)^2} + \frac{D}{(x+3)^2}$ Note that C=-4 and D=1

A1 Correct unsimplified answer $\frac{-4}{(2x+1)^2} + \frac{1}{(x+3)^2}$

Last 5 marks are identical

Alternative to 7(b) using product rule. First 3 marks are scored

M1 Writes f(x) as $5(2x+1)^{-1}(x+3)^{-1}$ and applies vu'+uv'. If the formula is quoted then it must be correct and an attempt made at both differentials. If it is not quoted you would expect to see an expression of the form
$5(2x+1)^{-1} \times Adapted\ (x+3)^{-1} + 5(x+3)^{-1} \times Adapted\ (2x+1)^{-1}$

M1 Applies the correct method for the chain rule on **both** $(2x+1)^{-1}$ and $(x+3)^{-1}$ resulting in the forms $(2x+1)^{-2}$(…..) and $(x+3)^{-2}(\ldots)$ respectively.

A1 A correct expression for the differential. It does not need to be simplified. A possible solution is
$5(2x+1)^{-1} \times -1(x+3)^{-2} + 5(x+3)^{-1} \times -1(2x+3)^{-2} \times 2$

Last 5 marks are identical

8.(a) M1 Using Pythagoras' Theorem to find R. Accept $R^2 = 2^2 + 3^2$.
If α has been found first accept $R = \frac{3}{sin\alpha}$ or $R = \frac{2}{cos\alpha}$

A1 R=√13 or awrt 3.61. Note many candidates just write this down and score M1A1 ($\pm\sqrt{13}$ is A0)

M1 For $tan\alpha = \pm\frac{3}{2}$ or $tan\alpha = \pm\frac{2}{3}$.
If R is used then ONLY accept $sin\alpha = \pm\frac{3}{their\ R}$ OR $\cos\alpha = \pm\frac{2}{their\ R}$

A1 AWRT 0.983. Note that awrt 56.3° would score A0.

(b) M1 Applies the product rule. If the rule is quoted it must be correct. Ie vu'**+**uv' and there must have been some attempt to differentiate both terms.
If the rule is not quoted (or implied by their working and the sight of u,v,u',v') only accept answers of the form $cos3x \times Ae^{2x} + e^{2x} \times \pm Bsin3x$. (*A* and *B* could be 1 for this mark)

A1 Any one term correct

A1 Both terms correct $cos3x \times 2e^{2x} + e^{2x} \times -3sin3x$

M1 Factorises out the e^{2x} term from their expression leaving $e^{2x}(\dots\dots\dots..)$

A1(*) CSO. This is a given answer. Minimum requirement is that candidate shows $e^{2x}(2cos3x - 3sin3x)$ **and** finishes off the 'proof' by writing $= R\,e^{2x}\cos(3x+\alpha)$ with *R* and α either as 'numbers' from part (a) or letters. Condone degrees. Even accept = $e^{2x}\,R\cos(3x+\alpha)$

(c) M1 For setting $\cos(3x + their\ \alpha) = 0$. Degrees are condoned for this mark

M1 For setting $3x + their\ \alpha = \frac{\pi}{2}$ and proceeding to x=… Accept for $\frac{\pi}{2}$ awrt 1.57.
Note the candidate must be working in radians

A1 Awrt 0.20. Do not accept degree equivalents, but a candidate who works in degrees and converts to radians achieving awrt 0.20 can score all 3 marks

Alt (c) The question states hence or otherwise so we can accept other solutions

M1 Sets their $cos3x \times Ae^{2x} + e^{2x} \times \pm Bsin3x$=0, cancels or factorises out the e^{2x} term and reaches an equation of the form $\tan(3x) = k$

M1 Solves in radians and achieves $x = \frac{\arctan k}{3}$

A1 awrt 0.20. Do not accept degree equivalents